CNS Institute for Physics Teachers

Title:	**Biocircuits: Signal Loss in Cable Transmission**
Original: **Revision:**	4 April 2009 15 April 2009
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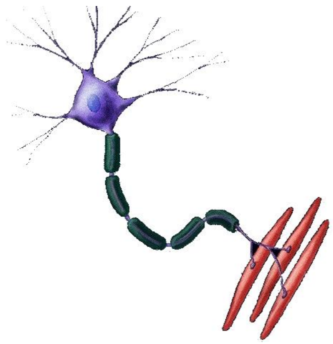
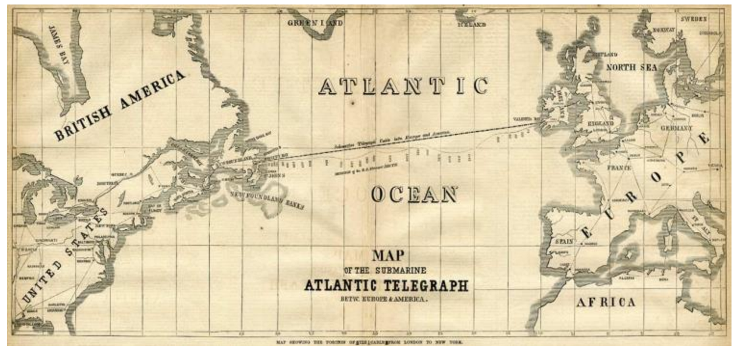
Appropriate Level:	9-12 Regents Physics, AP Physics B
Abstract:	In this lab students will investigate a basic (and important) electric circuit and show how it can be used to model nerve conduction. They will use Ohm's law to predict voltage and current flow in a simple voltage divider. Students will build a model electrical cable using a series of voltage dividers and experiment with strategies to reduce current loss and voltage decay along the cable. Students will use the cable they build to show how the same strategies they discovered are used in nature to solve the very important problem of nerve conduction and appreciate how the cable model predicts problems associated with an important disease, *Multiple Sclerosis*.
Time Required:	Two 40 minute lab periods
NY Standards Met:	Physical Setting: Physics 4.1l All materials display a range of conductivity. At constant temperature, common metallic conductors obey Ohm's Law*. 4.1o Circuit components may be connected in series* or in parallel.* Schematic diagrams are used to represent circuits and circuit elements. 4.1m The factors affecting resistance in a conductor are length, area, temperature, and resistivity.* 5.1d An object in linear motion may travel with a constant velocity* or with acceleration*.(Note: Testing of acceleration will be limited to cases in which acceleration is constant.) Living Environment 1.2a Important levels of organization for structure and function and whole organisms. 1.2f Cells have particular structures that perform specific jobs. These structures perform the actual work of the cell. Just as systems are coordinated and work together, cellparts must also be coordinated and work together. 1.2g Each cell is covered by a membrane that performs a number of important functions for the cell. These include:

	separation from its outside environment, controlling which molecules enter and leave the cell, and recognition of chemical signals. The processes of diffusion and active transport are important in the movement of materials in and out of cells.

Signal Loss in Cable Transmission





Pre-lab

Sarah flew out of bed that day. It was the first day of varsity volleyball practice and she could not wait to get to school. When she appeared at the breakfast table with a bruise on her arm she told her father she was, "Just clumsy I guess. You know, excited and all."

Eric didn't want to go to school. He had studied so hard and read his physics book over and over but he just could not remember.

"I'm tired. You go ahead." Everyone was excited to have the afternoon free to explore, but Stephanie was exhausted after the mornings walking tour of the Smithsonian Air and Space museum. She just had to rest.

People can fall, forget, or just be tired for many reasons. But for over 400,000 Americans, mostly young people, these are early signs of a serious condition that will change their lives. What causes Multiple Sclerosis? And, how does the disease affect so many different aspects of ordinary life?

Does this sound like a physics problem to you?

Multiple sclerosis is a nerve conduction problem. The body is sending out a signal for something to happen and the message just isn't getting there in time. This is similar to the problem a communications engineer might experience with a data signal in a computer cable. The data goes in one end but by the time it gets to the other end the signal has become weak and noisy. This problem was originally encountered by the telegraph companies in the middle of the 1800s when they tried to lay cables across the English Channel and later across the Atlantic Ocean. How could they design a cable that could carry the signal without significant loss of data and at a cost that the companies and their customers could afford? Enter Lord Kelvin. You may have heard the name associated with temperature as in: "zero Kelvin," *Absolute zero*, the coldest possible temperature. Lord Kelvin, he was a Baron whose real name was William Thompson, is about as

famous as they get in physics. Let's see how physics solved the problem of the transoceanic telegraph and how physicians used that knowledge to figure out why Sarah tripped over her Nikes® on the way to breakfast. And, by the way, remember that crayfish your brother couldn't catch at the lake last year. Lord Kelvin can explain that too.
Your challenge in this lab is to figure out what a disease of the nervous system, the transatlantic cable, and that illusive crayfish have to do with each other. Although you think of the nervous system and the crayfish as biological things, their properties that we are going to discuss in this lab are electrical. The basic electrical unit that unites these things is the voltage divider, which is a basic electrical circuit used to create a smaller output voltage from a larger input voltage.

Introduction to voltage dividers
When a voltage source with a fixed voltage output is available, one of the most basic problems in electronics is to reduce that voltage to a level appropriate for a particular task. For example, if you have a 12V battery and a lamp that can only handle 6V of energy, how can you reduce the voltage to an appropriate level. The simplest device available is the voltage divider. It consists of a voltage source and two resistors as shown in Figure 1.

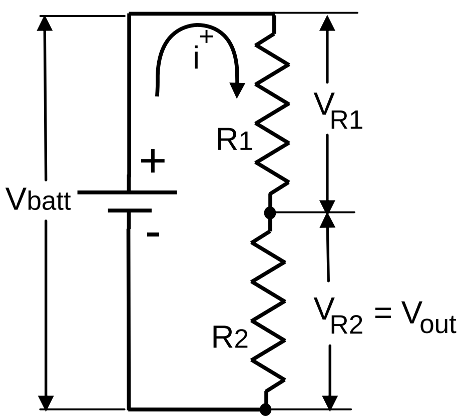


Figure 1. A simple voltage divider. Positive current (i^+) flows from the positive pole of the battery through two resistors in series to the negative pole of the battery.

According to Ohm's law, when current flows through a resistor, a voltage equal to i*R appears across the resistor. In Figure 1 this voltage is labeled V_{R1} for the voltage that appears across R_1 and V_{R2} for the voltage that appears across R_2. The total voltage drop across both resistors is equal to the voltage of the battery.

$$\boldsymbol{V_{batt} = V_{R_1} + V_{R_2}}$$

By convention, the voltage across R_2 is called V_{out} and this is the voltage the circuit designer is trying to achieve. The output voltage of the voltage divider is calculated using the formula

$$\boldsymbol{V_{out} = V_{batt}\left(\frac{R_2}{R_1 + R_2}\right)}$$

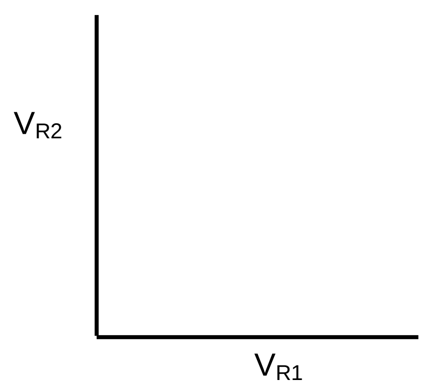
Use the formulas above to calculate V_{R1} and V_{R2} for each of the resistors listed in Table 1. Check your work by noting if $V_{batt} = V_{R1} + V_{R2}$.

Table 1: Voltage Divider Calculations

		1	2	3	4	5
A	V_{batt}	R_1 (Ω)	R_2 (Ω)	$R_2 / (R_1 + R_2)$ (a ratio)	$V_{R2} = V_{out}$ (V)	V_{R1} (V)
B	9V	1500	1500			
C	9V	1500	100			
D	9V	100	1500			
E	9V	100	200			

<u>Pre-lab Questions</u>

1. Plot the relationship between V_{R2} and V_{R1}.



2. Describe the relationship between the values in column 3 and V_{R2} (V_{out}).

3. Describe the relationship between the values in column 3 and V_{R1}.

4. To make V_{out} close to V_{batt}, how would you pick R_1 and R_2?

5. To make V_{out} much smaller than V_{batt}, how would you pick R_1 and R_2?

Figure 2 shows the voltage divider in a different orientation than you used in the Pre-lab.

In the rest of this lab, the voltage divider will look like the figure on the right. Study the two figures to be sure you agree they are the same.

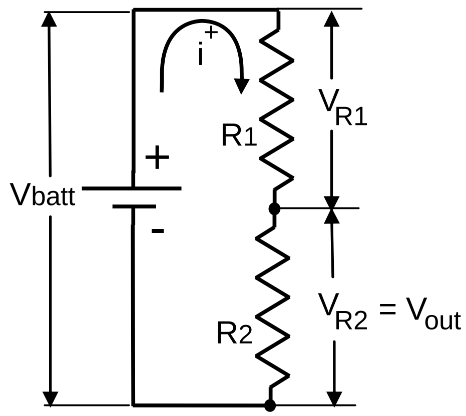


Figure 2: Two equivalent representations of a voltage divider.

In the appendix, at the end of the lab, Figure 10 shows the project board that will be used in this lab activity. Before beginning to work you should read the description and fasmiliarize yourself with the board.

Experimental Section

Purpose: In this lab you will investigate the problem of transmitting signals over long distances. You will discover that the solutions are governed by the laws of physics, whether the problem arises in biology or engineering. You will learn how a simple electronic device can be used to study the problem and make predictions that can be tested to better understand how both cables and nerves work.

Activity 1: Finding Physics in a Biology Problem

In the prelab you were introduced to a serious disease that is caused by a failure of nerves to properly carry signals throughout the body. Our success as animals depends on the ability of these nerves to respond rapidly to stimuli. You have probably had the experience of a painful stimulus causing you to quickly, without even thinking, pull your hand away. Figure 3 shows the nerves (electric wires) used to transmit signals between your limbs and central nervous system (CNS) in the withdrawal reflex. The little square in this diagram represents an area of skin on your hand containing receptors that are sensitive to painful stimuli. The part of a neuron that carries the signal over long distances is a long thin process called the axon. The signal travels along an axon toward the CNS. In the spinal cord, the signal is transferred to another neuron and travels along its axon back to the muscles in your arm causing the withdrawal. **The axons in this biological circuit are analogous to the wires in an electric circuit.** The term nerve is often used instead of axon. A nerve is composed of a bundle of many axons packaged together, much like an electrical cable is composed of a bundle of wires. In this lab we are studying axonal conduction.

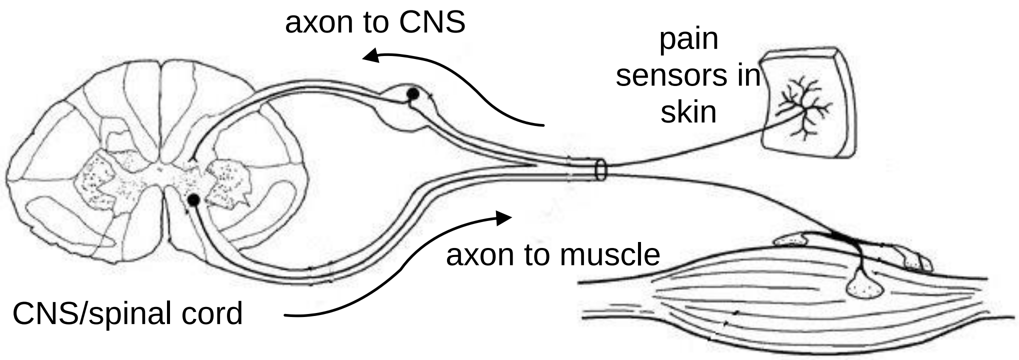


Figure 3. Neural circuit of the withdrawl reflex.

Materials:
- Wooden ruler or meter stick
- calculator

Procedure:
- Vertically hold a ruler between your fingers and record the position of your fingers on the ruler. This is x_1.
- Open and close your fingers as fast as you can. Record the new position of your fingers. This is x_2.
- $x_2 - x_1 = d$, the distance the ruler fell.
- Record your values in the Table 2.
- Calculate the time it took for your hands to open and close using the following equation. Solve for Δt in seconds and record this value in Table 2.

$$\boldsymbol{d} = \frac{1}{2}\boldsymbol{a}(\Delta \boldsymbol{t})^2$$

Where: d is the distance the rule moved (m)
a is the acceleration of gravity ($9.8 m/sec^2$)
Δt is the time it took to open and close your hand (sec)

- Estimate Δx, the total distance the signal must travel for you to open and close your hand by first measuring the distance from your fingers to the center of your spine at the level of your shoulder blades. What you are actually measuring are two signals: 0ne to release the ruler and the other to squeeze it. Double your value for Δx to account for this. Input this value into Table 2.

- Calculate the velocity using the equation in the table and enter the value in Table 2.

Table 2: Calculation of motor neuron transmission speed.

Initial ruler Position, x_1 (m)	Final ruler position, x_2 (m)	$d = x_2 - x_1$ (m)	Time, Δt (sec)	Δx = nerve pathway length, (m)	$v_{nerve} = \Delta x/\Delta t$ (m/sec)

1. Think about experiences in your daily life – or those of other creatures you know – and make a list of situations where fast response is crucial. Make a separate list of situations where fast reactions are less important.

Fast	Not so fast
______________________________	____________________________
______________________________	____________________________
______________________________	____________________________

2. Do you think the nerve cells, axons, and neural circuits used to perform all of the tasks in both of these lists have the same physical characteristics? How might they be different?

To better understand how signals are transmitted by nerve cells we need a simpler system to study. Once we have gained insights into signal transmission in electric wires we will return to look at some issues in biological signal transmission more thoroughly.

Activity 2: Modeling Signal Loss in the Transatlantic Cable

In this activity you will investigate the problem of transmitting electrical signals over long distances. In addition to modeling the performance of a cable, an engineering assessment of the cost/benefit analysis for cable design will be conducted.

Materials:
- project board
- digital multimeter
- 9V battery
- screw driver
- LED

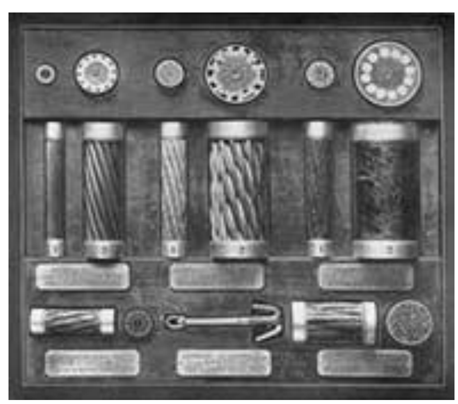


Figure 4. Transatlantic cables from 1858, 1865, and 1866.

• resistors (set of 5 each of 6 resistors shown in Table 3.)

Table 3: Resistor Values and Corresponding Color Codes

Core resistance, R_C, of the Cu wire	Red (400 Ω)	Blue (200Ω)	Green (100Ω)
Leak resistance, R_L, of the insulation	Yellow (7,500Ω)	Brown (2,200Ω)	Black (1,500 Ω)

Figure 5 shows the components modeled in the system: the core resistance of the wire and the leak resistance of the insulation.

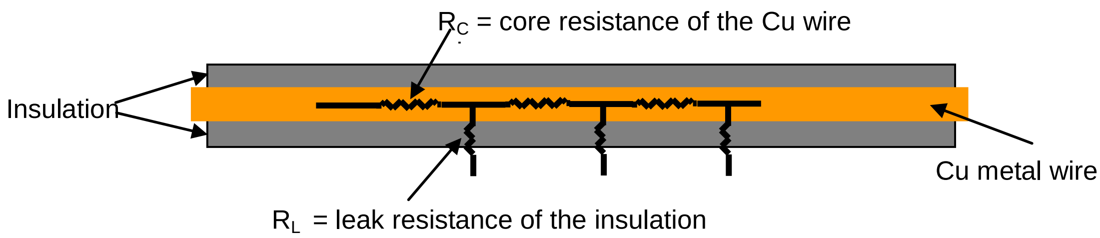


Figure 5: Cross section of an insulated metal wire.

<u>Core resistance of a wire</u>

When current flows through a wire it encounters a resistance that is a function of the material the from which the wire is made and the wire geometry. This resistance is called the 'core' resistance (R_{core} or R_C).

- A <u>high</u> core resistance means that the wire <u>is not</u> a good electrical conductor.
- A <u>low</u> core resistance means that the wire <u>is</u> a good electrical conductor.

<u>Leak resistance of insulation</u>

At the same time, some of the current will not flow along the wire, but will leak to ground. The better the wire is insulated, the less current will be leaked. The resistance of the insulation to this current leakage is called the 'leak' resistance (R_{leak} or R_L).

- A <u>well insulated</u> wire has a <u>high</u> leak resistance.
- A <u>poorly insulated</u> wire has a <u>low</u> leak resistance.

If the signal leaks to ground as it travels down the cable, then only part of the original signal will be available at the other end of the cable to do useful work. Figure 6 shows a voltage divider model of a wire that is similar to what Lord Kelvin developed to solve the problem of transmitting telegraph signals across the English Channel. You will test this voltage divider model using different combinations of core and leak resistances. Study the diagram and be sure you can identify the voltage dividers.

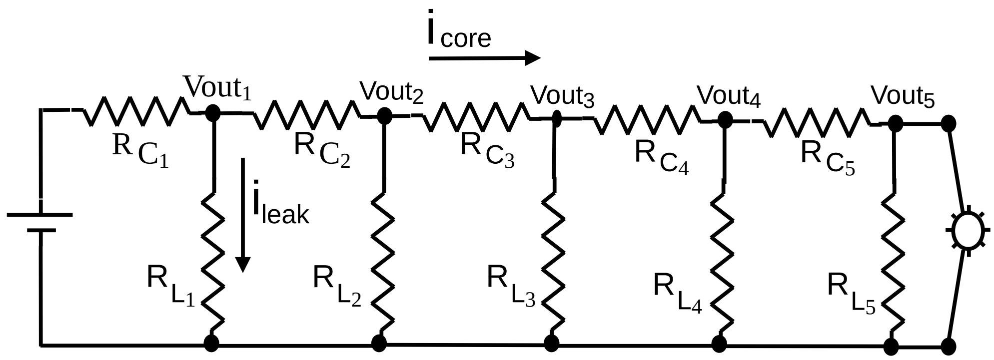


Figure 6: Circuit model for signal losses along a cable.

Note that the **output of the first divider in Figure 6 is the input to the second one and the output of the second divider is the input to the third one**, and so on. The output of the 5th segment is the voltage available at the end of the wire to light the LED.

Procedure:

- Refer to the appendix to learn about the Cable Transmission Project Board you will use for this activity
- Measure the actual voltage of your battery and record it in Table 4.
- On your project board, assemble the circuit shown in Figure 7, using $R_C = R_L =$ 1500 Ω (black). The left hand image is the circuit schematic and the right hand image shows how you build this circuit on your project board. The right hand image also shows how you measure the voltage across a resistor.

TIP Use a flat blade or pen tip to depress the lever with even pressure while inserting the resistor lead.

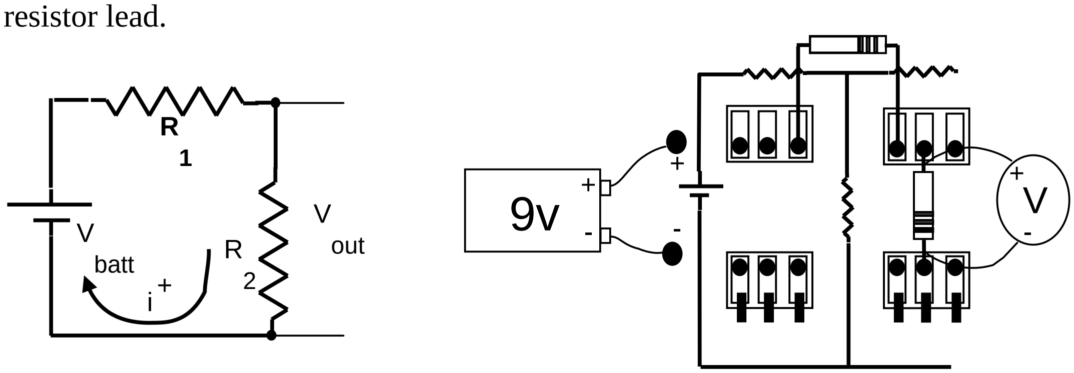


Figure 7. Voltage divider circuit and project board wiring diagram.

- Use the double clip leads to connect the 9V battery to the board as shown in Figure 7.

NOTE Disconnect the battery when you are not taking measurements.

- Measure the voltage across R_1 and R_2 (V_{out}) . Record these values in Table 4.
- Complete the data table by testing the other 3 combinations of resistors listed.
- Compare the values you measured with the values you calculated in the pre-lab. Do they agree with your predictions?

Table 4: Voltage Divider Data

V_{batt}	R_1 Ω color	R2 Ω color	V_{R1}	V_{R2} (V_{out})	$V_{R1} + V_{R2}$
	1500 (black)	1500 (black)			
	1500 (black)	100 (green)			
	100 (green)	1500 (black)			
	100 (green)	200 (blue)			

Modeling the Transatlantic Cable

- Assemble the circuit shown in Figure 6 using the resistor values shown in the first line of the Data Table below.
- All of the resistors used for R_C must be of the same value, because the wire is homogeneous along its length, and all of the resistors used for R_L must be the same, because the insulating material is also uniform along the length of the wire.
- To simplify the assembly of the cable the resistors have been color coded as shown in Table 4.
- Do not connect the LED at this time.
- Connect the battery and measure the voltage across the 5 leak resistors R_{L1} to R_{L5}. Enter the values in you measure in Table 3.
- After you have recorded your data connect the LED by touching its leads to the leads of the leak resistor R_{L5}. Record whether or not the LED lights in Table 3.

TIP: It isn't necessary to actually connect the LED to the board. You can just touch the LED leads to the resistor leads.

NOTE: LEDs have polarity. If the LED does not light, switch the leads to be sure current is flowing through the LED in the correct direction.

- Disconnect the battery and the LED.
- Repeat the procedure using the other values for R_C and R_L in the Data Table. Enter the values obtained into the Data Table.

Table 5: Voltage Divider Network Data

R_C	R_L	V_{out1}	V_{out2}	V_{out3}	V_{out4}	V_{out5}	LED (on/off)
LO (green)	MED (brown)						
LO (green)	LO (black)						
HI (red)	LO (black)						

Data for the six other possible configurations is plotted on the graph below in Figure 8.

- Plot your data for each of the combinations you tested onto Figure 8.

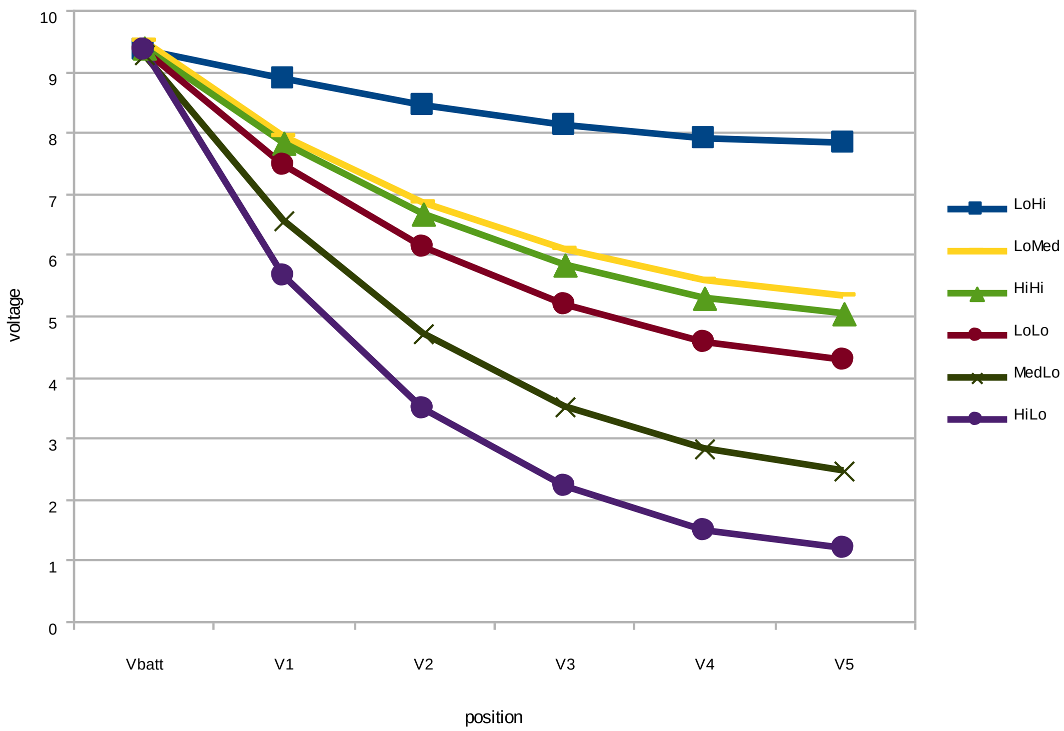


Figure 8: Voltage Divider Data for Different Resistor Combinations. The core resistance is listed first and the leak resistance is listed second.

Using your data and the data in Figure 8, determine which combinations of core and leak resistances lead to cables capable of lighting an LED. Summarize this information in Table 6 by writing “ON” if a particular combination of core and leak resistances can light the LED and writing “OFF” if it cannot.

Table 6: Summary showing which cables could light the LED.

		Core Resistance		
		High	Med	Low
Leak Resistance	Low			
	Med			
	High			

3. What trends do you notice in this table?

In the real world, engineers rarely have the luxury of building the best possible system since materials cost money and customers desire a working and reliable solution that is economically viable.

For the cable, the core resistance models the resistance of the wire itself. This follows the equation:

$$R = \rho L/A$$

4. For a given material (with resistivity ρ) and a given length wire (L), how do you make a wire with a low core resistance?

5. How do you make a cable with a high leak resistance?

6. Given this information, which cable configurations do you think are the least and most expensive to produce and why?

Historical data shows that the cost of the transatlantic cable was approximately £400/nautical mile in British Pounds, £. Table 7 has estimates for the cost of the various core and leak resistances/nautical mile of cable. The total cost per nautical mile is the sum of the cost of the core and leak resistances.

Table 7: Estimated material cost per nautical mile (in British Pounds) as a function of material quality

	HIGH	MED	LOW
Core resistance, R_C of the wire	£100.00	£200.00	£400.00
Leak resistance, R_L of the insulation	£200.00	£150.00	£50.00

- **Assume that your voltage divider circuit is a good model for the design and function of the transatlantic cable.**

7. Based on your LED model and the data in Table 6, determine the cost per nautical mile of each working model for the transatlantic cable. Show your work.

8. If you were the engineer responsible for the project, which combination of core and leak resistances would you decide to construct the cable from? Why?

Post Lab: Cable Conduction in Biology

In Activity 2 you modeled the transatlantic cable as a series of voltage dividers. The reason you did that on a project board with a picture of a nerve cell is that the same voltage divider model applies to signal transmission in the nervous system. To convert from metal cables to the biological system, we just need to redefine the variables:

Core resistance of an axon

When a signal travels along a nerve axon it encounters a resistance that is a function of nerve geometry.

- A high core resistance means that the nerve is thin (small cross sectional area)
- A low core resistance means that the nerve is a thick one (it has a large cross sectional area).

Leak resistance of an axon: the myelin sheath

Insulation of axons is provided by a layer of myelin. A schematic of a neuron with a myelinated axon is shown in Figure 9. *Myelin is used to insulate nerve axons whenever fast conduction speed is needed.* Not all axons are myelinated.

- A well insulated axon with a thick myelin layer has a high leak resistance.
- An axon with no myelin sheath has a low leak resistance.

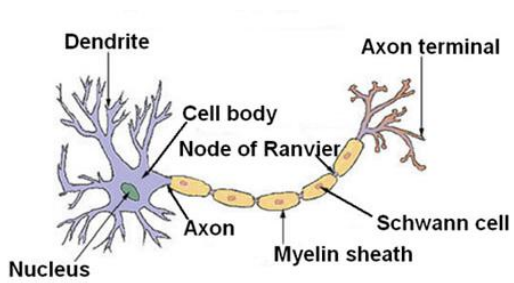


Figure 9. Schematic representation of a multipolar neuron with myelinated axon.

How fast is fast?

9. In nature, animals rely on fast reactions for survival. Based on your understanding of cables and nerves, speculate about strategies nature could use to increase the speed of conduction. Give your answer in terms of both R_C and R_L for axons. Make sure to explain what these values mean in terms of the axon structure.

10. Not all axons are myelinated. If myelin makes nerve signal transmission faster, why wouldn't all axons be myelinated?

11. Many invertebrate animals do have exceptionally fast neurons that are not insulated with myelin. Based on your understanding of cable conduction, what must be a common physical characteristic of these unmyelinated neurons to enable them to transmit signals so quickly?

12. How fast were your neurons? To get a better sense of this, convert your neuron speed from m/sec into miles/hour. Show your work.

Fun Fact: The ratio of the diameter to the length of the transatlantic cable is similar to that of a long alpha motor neuron. Engineering produced the same solution as biology!

Here are some examples of unmyelinated axons in invertebrates:

- The squid contracts its mantle providing a jet powered escape mechanism using a signal from an axon known as the squid giant axon. It's so big you can dissect it without using a microscope.
- The cockroach has a pair of hairs on its tail end called cerce. Large unmyelinated axons carry information from these hairs to its brain about air currents coming from behind it.
- The crayfish (remember the crayfish that got away) uses rapid beats of its tail to dart away for cover. This movement is stimulated by large unmyelinated axons.

- Which of these neurons do you think would have myelinated axons? Explain your reasoning.

Nerve Type	Axon Diameter (μm)	Signal Velocity (m/s)	Myelinated? Yes or No	Explanation
Squid Giant Axon	500	19		
α-motor neuron (skeletal muscle)	10	120		
Sensory neuron (stimulus response)	1-5	30		
c-fiber pain	0.2 – 1.5	0.5 – 2.0		

(not sudden)				

Fun Fact: The neurons in your brain have myelinated axons to assure fast conduction. If they were not, to achieve the same transmission speed, the nerves would have to be much larger in diameter. In fact, your head would have to be the size of a beach ball!

Let’s go back to the Pre-lab:

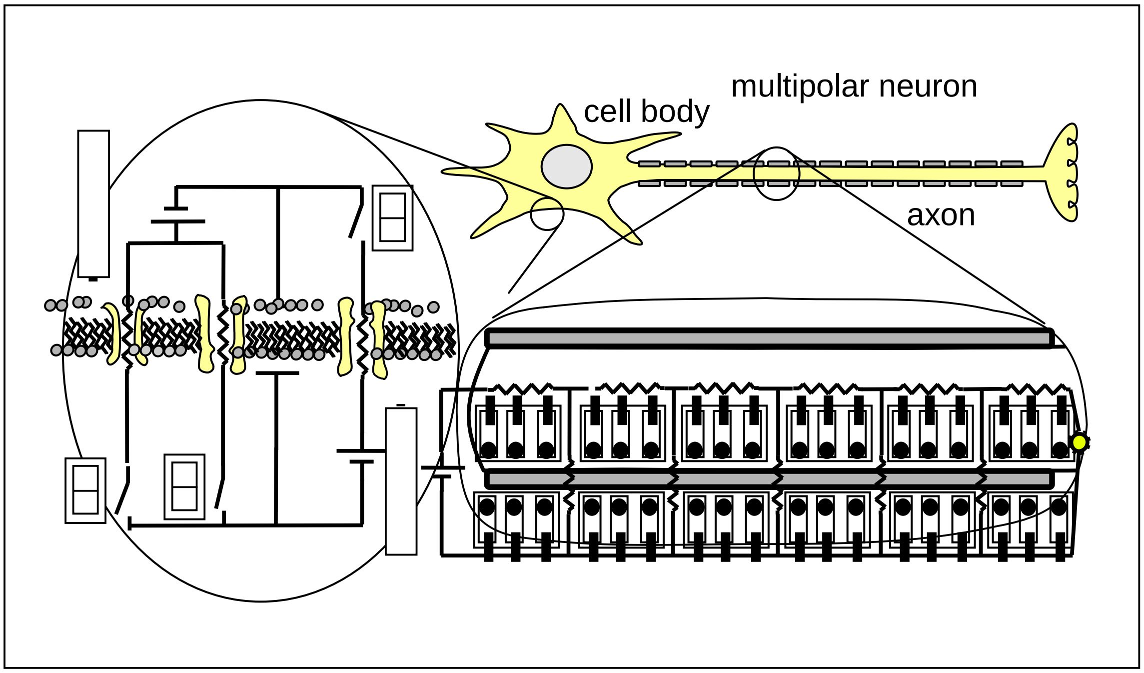
Multiple Sclerosis

What happens in your body if you have Multiple Sclerosis? This is an autoimmune disease, which means your body attacks itself. In multiple sclerosis the core resistance of your nerves is not affected.

13. According to what you have learned in this lab, describe the type of damage to Sarah, Eric, and Stephanie's nervous systems that occurred as a result of having Multiple Sclerosis.

14. Science has not figured out how to cure this disease. How could stem cell research lead to a potential cure for this disease?

Figure 10. The Cable Transmission Project Board



Even though most of the work done in lab will concern conventional electric circuits, the project board used was designed to emphasize the connection between physics and biology.

There are 3 main areas on the board.

- At the top and to the right is a diagram of a multipolar neuron. This is the type of neuron that is used to carry signals to muscles. When activated, these neurons cause muscles to contract.
- On the left side of the board in the large circle is a detailed diagram of a section of the cell body and the circuit that describes the action potential. The cell body is where the neuron makes the decision as to whether or not it should fire an action potential. It will not be used in this activity.
- At the bottom and to the right is a detailed diagram of a segment of axon. The axon is the structure that transmits the action potential generated in the cell body and carries it to the muscle. On the left side of this diagram is a symbol for a battery. This is where you will attach your signal source, a 9V dry cell battery. On the far right is an LED. The LED will be used as the signal that the circuit you've constructed works.

<u>Study the diagram and see if you can find any voltage dividers in the circuit between the battery and the LED.</u>